

## Effect of Vacuum Energy Accretion on Primordial Black Holes in Brans-Dicke theory

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**Abstract :** In this paper, we investigate the evolution of primordial black hole (PBH) in vacuum-dominated era within Brans-Dicke cosmology. We consider the accretion of vacuum energy by PBHs and found that vacuum energy accretion efficiency should be less than 0.61. We also study the evaporation of PBHs where we conclude that larger the value of accretion efficiency longer live the PBHs. We also find that PBHs evaporate at a quicker rate in Brans-Dicke theory compared with Standard Cosmology [1].

### 1. Introduction

In reference [1], by taking General Theory of Relativity as the theory of gravity, it is shown that during vacuum dominated era, the accretion of vacuum energy increases the mass of primordial black holes. However, the accretion is possible only upto a critical time  $t_c$ , which depends on accretion efficiency and formation time, and the life span of PBH increases if it lives beyond this critical time  $t_c$ . Here we try to extend this work by changing the theory of gravity from GTR to scalar-tensor theory like Brans-Dicke theory. Due to the time variation of  $G$  in Brans-Dicke theory, the scale factor takes a different form and it controls the PBH evolution in a different manner than GTR. So it is worthwhile to take a study on PBH evolution in vacuum dominated era within Brans-Dicke theory what we try to do in this paper.

Einstein's General Theory of Relativity (GTR) [2] is based on a pure tensor theory of gravity where gravitational constant is taken as a time-independent quantity. But Brans Dicke(BD) theory [3] is a scalar-tensor theory of gravity where the gravitational constant is a time-dependent quantity. BD theory is the simplest extension over GTR through the introduction of a time-dependent scalar field  $G(t) \propto \phi^{-1}(t)$ , where the scalar field  $\phi(t)$  couples to gravity with a coupling parameter  $w$  known as the BD parameter. Interestingly BD theory can be transformed to GTR in the limit  $w \rightarrow \infty$  [4, 5]. Solar system observations require  $w > 10^4$  [6]. Also BD type model can be regarded as the lowest limit of Kaluza-Klein and String theories [7, 8, 9]. Again BD theory explains many cosmological phenomena such as inflation [10, 11], early and late time behaviour of the universe [12, 13], cosmic acceleration and structure formation [14], coincidence problem [15] and problems relating to black holes [16].

Primordial Black Holes (PBHs) are those black holes which are formed in the early universe through variety of mechanisms such as inflation [17, 18], initial inhomogeneities [19, 20], phase transition and critical phenomena in gravitational collapse [21, 22, 23, 24, 25, 26, 27], bubble collision [28] or the decay of cosmic loops [29, 30]. A comparison of cosmological density of the universe with the density associated with a black hole, at any time after Big Bang, shows that formation mass of PBH would have same order as that of horizon mass. Thus PBH could span wide mass range starting from Planck mass  $10^{-5}g$  to more than  $10^{15}g$ . From Hawking's point of view black holes emit thermal radiation quantum mechanically [31]. So black holes will evaporate depending upon their formation masses. Smaller the mass of PBHS, quicker they evaporate. As density of a black hole varies inversely with its mass, high density which is possible in the early universe, is required to form lighter black holes. So PBHs are the only black holes whose masses could be so small that they became able to evaporate completely by the present epoch through Hawking evaporation [31]. Early evaporating PBHs could account for baryogenesis [32, 33, 34, 35, 36], in the universe. On the other hand, longer lived PBHs could act as seeds for structure formation [37, 38, 39, 40, 41, 42] and could also form a significant component of the dark matter [43, 44, 45, 46]. Again in refs [47, 48, 49], it is concluded that PBHs could take comparatively more time to evaporate due to accretion of radiation which makes them long lived.

Accordingly the standard picture of cosmology, the universe is radiation-dominated in the very beginning of its evolution and now it is matter-dominated. This gives the decelerated expansion of universe through its evolution. But the

observations of distant measurements to type Ia Supernovae (SNIa) [50, 51, 52] indicate that the expansion of universe is accelerating in the present epoch and that nearly two-third of the critical energy density exists in a dark energy component with a large negative pressure and of unknown composition. The simplest possibility of dark energy is vacuum energy. Recent data shows that dark energy occupies 68.3% of universe and 31.7% is occupied by some other matter. SNIa observations also provide the evidence of a decelerated universe in the recent past with transition from decelerated to accelerated occurring at redshift  $z_{q=0} \approx 0.5$  [53, 54]. So the vacuum energy should be started from  $z_{q=0} \approx 0.5$  i.e.  $t_{q=0} \approx 0.5t_0$ . The equation of state parameter  $\gamma = -1$  is the most acceptable candidate for dark energy.

In this work, we examine the evolution of PBHs only in vacuum dominated era using Brans-Dicke theory. In our study, we consider vacuum energy accretion by PBHs and its effect on their evaporation. We also present a comparison between our results and the corresponding results of Standard Cosmology.

## **2. Solutions of scale factor $a(t)$ and gravitational constant $G(t)$ in vacuum dominated era**

For a spatially flat ( $k = 0$ ) Friedmann-Robertson-Walker universe with scale factor  $a(t)$ , the first Friedmann equation using Brans-Dicke theory in vacuum dominated era is

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{\phi}}{a\phi} - \frac{w\dot{\phi}^2}{6\phi^2} = \frac{8\pi\rho_v}{3\phi} \quad (1)$$

where  $\rho_v$  is the vacuum energy density.

The total energy conservation equation is given by

$$\dot{\rho} + 3H(\gamma + 1)\rho = 0 \quad (2)$$

assuming a perfect fluid equation of state  $p = \gamma\rho$ .

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter,  $\rho$  is the total energy density,  $\gamma (= -1)$  is the equation of state parameter for vacuum energy. So equation (2) gives vacuum energy density  $\rho_v$  is a constant.

From our previous paper [55], we have obtained the time dependent gravitational constant for vacuum dominated era by matching the time dimension of each term of Friedmann equation (1) as

$$G(t) = G_0 \left( \frac{t_0}{t} \right)^2 \quad (3)$$

where  $G_0$  is the present value of the gravitational constant and  $t_0$  is the present age of the universe.

Using equation (3), the above Friedmann equation can be expressed as

$$\frac{\dot{a}^2}{a^2} + \frac{2}{t} \frac{\dot{a}}{a} - \left( \frac{8\pi G_0}{3} \frac{t_0^2}{t^2} \rho_v + \frac{2}{3} \frac{w}{t^2} \right) = 0 \quad (4)$$

The solution of this equation gives

$$a(t) \propto t^{\left( -1 + \sqrt{1 + \frac{8\pi G_0}{3} t_0^2 \rho_v + \frac{2}{3} w} \right)} \quad (5)$$

where equations (3) and (5) give the expressions for  $G$  and scale factor  $a(t)$ .

### 3. Accretion of Vacuum energy by PBH

PBH mass can be changed by accumulating vacuum energy. The mass of PBH increases due to accretion with a rate given by

$$\dot{M}(t)_{acc} = 4\pi f_{vac} R_{BH}^2 \rho_v \quad (6)$$

where  $R_{BH} = 2GM$  is the black hole radius,  $\rho_v = \Omega_{\Lambda}^0 \rho_{critical}$  with  $\Omega_{\Lambda}^0$  is the present value of the vacuum energy density parameter and  $f_{vac}$  denotes the vacuum energy accretion efficiency.

Now using the above expressions of  $R_{BH}$  and  $\rho_v$  with equation (3), we can write equation (6) as

$$M(t)_{acc} = 16\pi G_0^2 \left( \frac{t_0}{t} \right)^4 f_{vac} M_{acc}^2 \Omega_{\Lambda}^0 \rho_{critical} \quad (7)$$

On integration, equation (7) gives

$$M(t)_{acc} = M_i \left[ 1 + \frac{16}{3} \pi G_0^2 t_0^4 f_{vac} \Omega_{\Lambda}^0 \rho_{critical} M_i (t^{-3} - t_i^{-3}) \right]^{-1} \quad (8)$$

where  $M_i$  is the initial mass of PBH formed at time  $t_i$  and using the variation of horizon mass with time, we write  $M_i = G^{-1}t_i = \left[ G_0 \left( \frac{t_0}{t_i} \right)^2 \right]^{-1} t_i$ .

Now the equation (8) becomes

$$M(t)_{acc} = M_i \left[ 1 + \frac{16}{3} \pi G_0 t_0^2 \Omega_\Lambda^0 \rho_{critical} f_{vac} \left\{ \left( \frac{t_i}{t} \right)^3 - 1 \right\} \right]^{-1} \quad (9)$$

Again using the numerical values of different quantities like  $G_0 = 6.67 \times 10^{-8}$  dyne -  $cm^2/g^2$ ,  $\rho_{critical} = 1.1 \times 10^{-29}$  g/cm<sup>2</sup>,  $t_0 = 4.42 \times 10^{17}$  and  $\Omega_\Lambda^0 = 0.683$ , we obtain

$$M(t)_{acc} = M_i \left[ 1 + 1.639 f_{vac} \left\{ \left( \frac{t_i}{t} \right)^3 - 1 \right\} \right]^{-1} \quad (10)$$

This equation gives the mass of PBH due to accretion only. We can also write equation (10) as

$$M(t)_{acc} = M_i \left[ 1 - 1.639 f_{vac} \left\{ 1 - \left( \frac{t_i}{t} \right)^3 \right\} \right]^{-1} \quad (11)$$

For validity of equation (11)

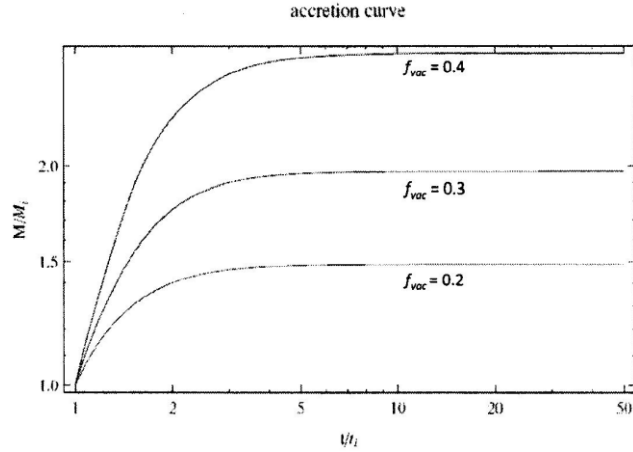
$$f_{vac} < \frac{1}{1.639 \left\{ 1 - \left( \frac{t_i}{t} \right)^3 \right\}} \quad (12)$$

For large time  $t, t_i/t \rightarrow 0$  and we get

$$f_{vac} < \frac{1}{1.639} \approx 0.61 \quad (13)$$

The variation of PBH mass with time due to accretion only is shown in Fig.1.

The Fig.1 shows that at a particular time the mass of PBH increases with increase in accretion efficiency and for a particular accretion efficiency mass saturates after a particular time.



**Fig. 1:** Variation of PBH mass with time for different accretion efficiencies.

#### 4. PBH Evaporation in Vacuum Dominated Era

Due to Hawking radiation [31], the rate of decrease of PBH mass is given by

$$\dot{M}_{evap} = -4\pi R_{BH}^2 a_H T_{BH}^4 \tag{14}$$

where  $a_H$  is the Stefan-Boltzman constant multiplied with number of degrees of freedom available for radiation and  $T_{BH} = \frac{1}{8\pi GM}$  is the Hawking temperature.

Using the solution of  $G(t)$  with the above expressions of  $R_{BH}$  and  $T_{BH}$  in vacuum dominated era we can write the rate of decrease of PBH mass as

$$\dot{M}_{evap} = -\frac{a_H}{256\pi^3} \frac{1}{G_0^2 \left(\frac{t_0}{t}\right)^4} M^2 \tag{15}$$

We, now, study the evolution of PBH by considering both accretion and evaporation in vacuum dominated era.

In this case, the rate of change of PBH mass is given by

$$\dot{M}(t)_{PBH} = 16\pi G_0^2 \left(\frac{t_0}{t}\right)^4 f_{vac} \Omega_{\Lambda}^0 \rho_{critical} M_{PBH}^2 - \frac{a_H}{256\pi^3} \frac{1}{G_0^2 \left(\frac{t_0}{t}\right)^4} M_{PBH}^2 \tag{16}$$

This equation can not be solved analytically. So we use numerical method to solve it.

The evaporation of PBH for different accretion efficiencies is shown in Fig. 2. The Fig.2 shows that with increase in accretion efficiency life-time of PBH increases.

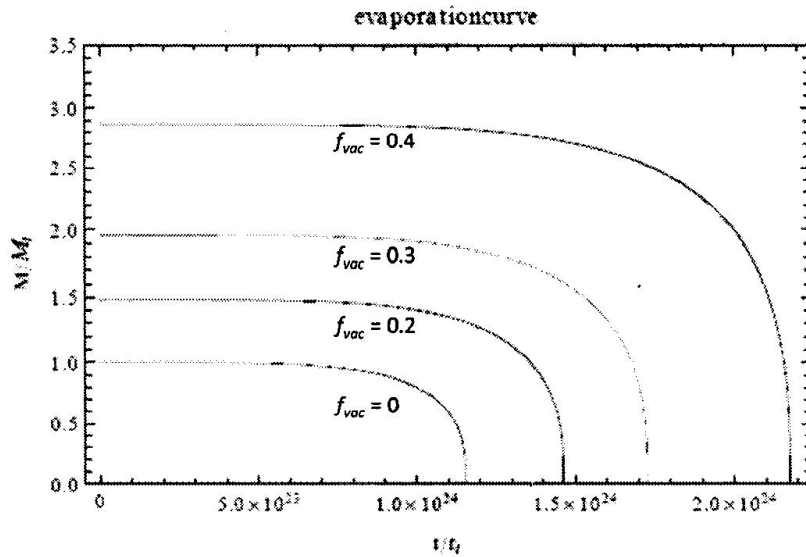


Fig. 2: Evaporation of a PBH having formation time  $t_i = 2.21 \times 10^{17}$ s for different accretion efficiencies.

## 5. Comparison with Standard Cosmology

Comparison of the above results with those of Standard Cosmology [1] shows many interesting differences. In Standard Cosmology vacuum energy accretion is only possible upto a critical time  $t_c$  which depends on accretion efficiency and formation time and beyond which accretion stops while in Brans-Dicke theory vacuum energy accretion is possible for all values of time ( $t$ ) eventhough the accreting mass saturates beyond certain time depending on accretion efficiency. We also observe that in BD theory the accretion of vacuum energy by PBH occurs at a smaller rate leading to quicker evaporation compared with Standard Cosmology [1]. Further, the evaporation time in BD theory is found to depend upon vacuum energy accretion efficiency whereas in Standard Cosmology it is independent of the accretion efficiency.

The evaporation time of PBH formed at  $t_i = 2.21 \times 10^{17}s$  for both Brans-Dicke (BD) theory and Standard Cosmology (SC) are shown in the table-1.

**Table 1:** Evaporation times of PBHs which are formed at starting of vacuum-dominated era for different accretion efficiencies.

$t_i = 2.21 \times 10^{17}s$		
$f_{vac}$	$(t_{evap})$	$(t_{evap})_{SC}$
0	$2.547 \times 10^{41}s$	$3.695 \times 10^{59}s$
0.2	$3.233 \times 10^{41}s$	$1.867 \times 10^{71}s$
0.4	$4.831 \times 10^{41}s$	$1.867 \times 10^{71}s$
0.6	$3.012 \times 10^{42}s$	$1.867 \times 10^{71}s$

## 6. Conclusion

Here we study the evolution of primordial black hole (PBH) in vacuum-dominated era within Brans-Dicke cosmology. First we obtain the expressions for scale factor  $a(t)$  and the gravitational constant  $G(t)$  in vacuum-dominated era. Then we study the accretion of vacuum energy by PBHs where we notice that the accretion efficiency has an upperbound of 0.61. We, then, find that PBH mass increases with accretion efficiency leading to longer life as the accretion efficiency takes higher values. A comparison with results of Standard Cosmology [1] reveals that life-time of PBHs are many orders of magnitude smaller as a result of slow accretion rate.

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